ON THE QUESTION OF THE STABILITY OF POWDER COMBUSTION IN A HALF-CLOSED SPACE

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The influence of gas temperature perturbations on the stability of powder combustion in a rocket chamber is investigated theoretically on the basis of the Zel'dovich-Novozhilov theory of powder combustion. The influence of the bow space adjacent to the burning channel is also examined.

The stability of the stationary powder combustion mode in a half-closed space has been investigated earlier in [1, 2]. The change in gas temperature in the chamber was hence not taken into account for a rapid change in pressure. Taking account of the influence of the gas temperature fluctuations was examined in [3]. However, underlying the investigation in [3] was a specific stationary fuel combustion model, which predetermines the narrowness of the application of the results obtained. This paper is based on the nonstationary theory of combustion developed in [4, 6] and relying on the experimental dependences of the combustion velocity on the parameters.

1. Influence of Gas Temperature Fluctuations

The stationary combustion velocity in the adiabatic mode depends only on the pressure and initial temperature of the powder. In real combustion cases the heat transfer from the flame zone results in a reduction of the maximum temperature versus the adiabatic combustion temperature, which results in a reduction in the combustion velocity and in the presence of propagation limits. Hence, it is assumed below that the stationary combustion velocity $u°$ and the surface temperature $T_1°$ are functions of the pressure p, the initial powder temperature T_0, and the gas temperature in the chamber T_2:

$$u^\circ = u^\circ(p,\ T_0,\ T_2),\quad T_1^\circ = T_1^\circ(p,\ T_0,\ T_2) \tag{1.1}$$

According to Ya. B. Zel'dovich [4, 5], by using the stationary connection between the temperature gradient on the surface $f°$, the combustion velocity, the surface temperature, and the initial temperature

$$\varkappa f^\circ = u^\circ(T_1^\circ - T_0) \tag{1.2}$$

it is possible to go from a dependence of the kind (1.1) to dependences such as

$$u = u(p,\ f,\ T_2),\quad T_1 = T_1(p,\ f,\ T_2) \tag{1.3}$$

which are valid even under nonstationary conditions. Here $\varkappa$ is the coefficient of powder temperature conduction.

Introducing the parameters k, ν, q, r, μ, s which characterize the dependence of the combustion velocity and surface temperature on the pressure, initial temperature, and gas temperature in the stationary mode

$$k = (T_1^\circ - T_0)\left(\frac{\partial \ln u^\circ}{\partial T_0}\right)_{p,T_2},\quad \nu = \left(\frac{\partial \ln u^\circ}{\partial \ln p}\right)_{T_0,\ T_2},\quad q = \left(\frac{\partial \ln u^\circ}{\partial \ln T_2}\right)_{p,\ T_0}$$

$$r = \left(\frac{\partial T_1^\circ}{\partial T_0}\right)_{p,\ T_2},\quad \mu = \frac{1}{T_1^\circ - T_0}\left(\frac{\partial T_1^\circ}{\partial \ln p}\right)_{T_0,\ T_2},\quad s = \frac{1}{T_1^\circ - T_0}\left(\frac{\partial T_1^\circ}{\partial \ln T_2}\right)_{p,\ T_0}$$

formulas connecting the derivatives of the combustion velocity and the surface temperature under stationary and nonstationary conditions are easily obtained from (1.1)-(1.3):

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$$\left(\frac{\partial \ln u}{\partial \ln p}\right)_{f,\,T_2} = \frac{\nu(r-1)-k\mu}{k+r-1}, \qquad \left(\frac{\partial \ln u}{\partial \ln f}\right)_{p,\,T_2} = \frac{k}{k+r-1}$$

$$\left(\frac{\partial \ln u}{\partial \ln T_2}\right)_{p,\,f} = \frac{q(r-1)-ks}{k+r-1}, \qquad \frac{1}{T_1^\circ - T_0}\left(\frac{\partial T_1}{\partial \ln p}\right)_{f,\,T_2} = \frac{\mu(k-1)-\nu r}{k+r-1} \tag{1.4}$$

$$\frac{1}{T_1^\circ - T_0}\left(\frac{\partial T_1}{\partial \ln f}\right)_{p,\,T_2} = \frac{r}{k+r-1}, \qquad \frac{1}{T_1^\circ - T_0}\left(\frac{\partial T_1}{\partial \ln T_2}\right)_{p,\,f} = \frac{s(k-1)-qr}{k+r-1}$$

The heat conduction equation

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial \xi^2} - v\frac{\partial \theta}{\partial \xi} \qquad (0 \geqslant \xi > -\infty) \tag{1.5}$$

with the conditions

$$\theta(0, \tau) = \vartheta, \qquad \theta(-\infty, \tau) = 0$$

is valid for a single inertial domain, a heated powder layer, where θ, ϑ, v, ξ, τ are, respectively, the dimensionless temperature in the powder, the surface temperature, the combustion velocity, the space coordinate, and the time

$$\theta = \frac{T - T_0}{T_1^\circ - T_0}, \quad \vartheta = \frac{T_1 - T_0}{T_1^\circ - T_0}, \quad v = \frac{u}{u^\circ}, \quad \xi = \frac{u^\circ}{\varkappa}x, \quad \tau = \frac{(u^\circ)^2}{\varkappa}t$$

Limiting ourselves to an investigation of the stability of the stationary combustion mode relative to small perturbations, we obtain in a linear approximation

$$\eta = \frac{p}{p^\circ} = 1 + \eta_1\psi(\tau), \qquad \zeta = \frac{T_2}{T_2^\circ} = 1 + \zeta_1\psi(\tau), \qquad v = 1 + v_1\psi(\tau),$$

$$\theta = [1 + \theta_1(\xi)\psi(\tau)]\exp\xi, \quad \vartheta = 1 + \vartheta_1\psi(\tau), \quad \varphi = \frac{\partial\theta(0,\tau)}{\partial\xi} = 1 + \varphi_1\psi(\tau)$$

where ζ is the dimensionless temperature in the chamber, $\psi(\tau)$ is some function of the time, η_1, ζ_1, v_1, θ_1, ϑ_1, ζ_1 are much less than one in absolute value. We hence obtain the following equation from (1.5) for the temperature correction

$$\theta_1'' + \theta_1' - \frac{\theta_1}{\psi}\frac{d\psi}{d\tau} - v_1 = 0 \tag{1.6}$$

with the conditions

$$\theta_1(0) = \vartheta_1, \qquad \theta_1(-\infty) = 0$$

and the relations

$$(k + r - 1)\, v_1 = [\nu(r-1) - k\mu]\,\eta_1 + k\varphi_1 + [q(r-1) - ks]\,\zeta_1$$
$$(k + r - 1)\,\vartheta_1 = [\mu(k-1) - \nu r]\,\eta_1 + r\varphi_1 + [s(k-1) - qr]\,\zeta_1 \tag{1.7}$$

result from (1.3) taking account of (1.4).

The pressure in the chamber is subjected to the mass conservation law

$$V_0\frac{d}{dt}\left(\frac{p}{RT_2}\right) = \rho_0\sigma u - \frac{A_0}{\sqrt{RT_2}}F_*p \tag{1.8}$$

for a variable gas temperature.

Here V_0 is the free space, σ is the combustion surface, ρ_0 is the powder density, F_* is the area of the critical nozzle section. A_0 is a constant, and R is the gas constant. Going over to dimensionless variables in (1.8), we obtain in a linear approximation

$$\chi(\eta_1 - \zeta_1)\frac{d\psi}{d\tau} = \left(v_1 - \eta_1 + \frac{\zeta_1}{2}\right)\psi \tag{1.9}$$

where χ is the ratio between the relaxation times of the heated powder layer t_2 and of the chamber t_1

$$\chi = \frac{t_1}{t_2}, \qquad t_2 = \frac{\varkappa}{(u^\circ)^2}, \qquad t_1 = \frac{V_0}{A_0F_*\sqrt{RT_2^\circ}}$$

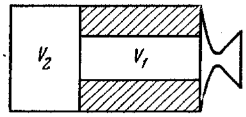

Fig. 1

Let us use the energy equation, written

$$V_0 \frac{d}{dt}(\rho RT_2) = \left(\rho_0 \sigma u - \frac{A_0}{\sqrt{RT_2}} F_* p\right) nRT_2 \qquad (1.10)$$

to close the system (1.6), (1.7), (1.9).

Here ρ is the gas density in the chamber, n is the adiabatic index. To write (1.10) more rigorously, a member taking account of heat losses in the chamber should be added to the right side. The heat losses have been taken into account approximately in (1.10), by introducing the effective adiabatic index. An increase in n results in a rise in the heat losses. From (1.10) we have in a linear approximation

$$\frac{\chi}{n} \eta_1 \frac{d\psi}{d\tau} = \left(v_1 - \eta_1 + \frac{\zeta_1}{2}\right) \psi \qquad (1.11)$$

Let us assume that small perturbations $\sim \exp(\gamma\tau)$ are imposed on the stationary combustion mode. Then the mode stability will be determined by the law of the real part of the frequency $\Omega = \mathrm{Re}\,\gamma$. Stability may hence be lost in going through the critical mode by both a continuous and a jump change in the damping decrement.

In order to investigate the disruption of stability of the first kind on the stability boundary, let us assume $\psi(\tau) = \exp(i\gamma\tau)$ (γ is real). Then the solution of (1.6) is found easily and a relationship between v_1, φ_1, and ϑ_1 obtained earlier in [7] results therefrom. Considering this relationship together with (1.7), (1.9), (1.11) as a system of algebraic equations in η_1, ζ_1, v_1, ϑ_1, φ_1, we obtain that the necessary condition for compatibility of the equations is that the system determinant equal zero. Expanding the determinant and equating the real and imaginary parts to zero, we arrive at two equations

$$c\left(a - \frac{n+1}{2n} c\right) - d\left(b + \frac{n+1}{2n} d\right) = 0 \qquad (1.12)$$

$$\chi = \frac{n}{\gamma(c^2 + d^2)} \left[d\left(a - \frac{n+1}{2n} c\right) + c\left(b + \frac{n+1}{2n} d\right)\right]$$

where

$$a = \nu + \frac{\gamma S_1}{2R_1}(\nu r - k\mu) + n\left[q + \frac{\gamma S_1}{2R_1}(qr - ks)\right]$$

$$b = R_1[\nu r - k\mu + n(qr - ks)], \qquad c = 1 + \frac{r\gamma S_1}{2R_1} - kS_1$$

$$d = \frac{kS_1}{2R_1} - rR_1, \qquad S_1 = 1 - \frac{R_1}{\gamma}, \qquad R_1 = 2^{-3/2}[(16\gamma^2 + 1)^{1/2} - 1]^{1/2}$$

Having been given a specific γ, we determine the value of k from the first equation in (1.12) for given physicochemical parameters of the powder, and χ corresponding to the stability boundary from the second equation. Numerical computations using (1.12) are presented below.

The singular case $\gamma = 0$ is not contained in (1.12). Putting $\psi(\tau) = \exp(\Omega\tau)$ ($\Omega \ll 1$) in order to investigate it, and reasoning analogously, it can be shown that the loss of stability for $\gamma = 0$ occurs in the case

$$\nu = \frac{n + 1 - 2(n-1)q}{2n} < 1$$

Therefore, the stationary combustion mode can turn out to be unstable even for $\nu < 1$ when taking account of the dependence of the combustion velocity on the gas temperature.

To investigate the possibility of a jump disruption of stability, we should put $\psi(\tau) = \exp(\pm\Omega\tau)$ ($\Omega \gg 1$). Then, as in [2], it can be shown that for a variable surface temperature this kind of loss of stability is not generally realized, but occurs at k = 1 for a constant temperature.

2. Dependence of the Combustion Stability in the Presence of a Bow Space (Fig. 1)

A pressure change in the channel for a constant temperature in the chamber and taking account of the gas overflowing into the bow space-channel system is subject to the balance equation

$$\frac{V_1}{RT_2^\circ} \frac{dp_1}{dt} = \rho_0 \sigma u - AF_* p_1 - M \qquad (2.1)$$

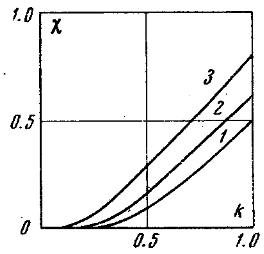

Fig. 2

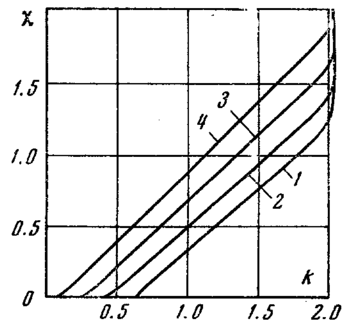

Fig. 3

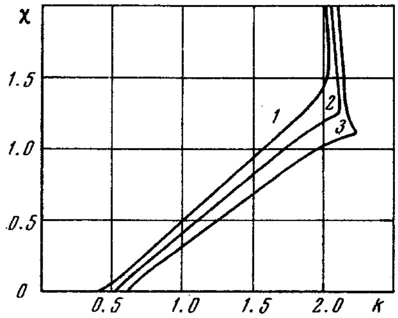

Fig. 4

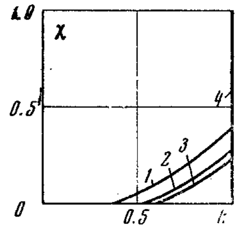

Fig. 5

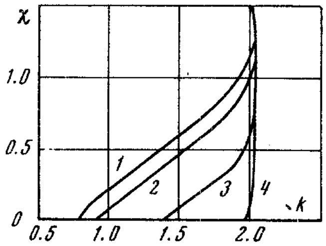

Fig. 6

where the rate of pressure change in the bow space is found from the equation

$$\frac{V_2}{RT_2^\circ}\frac{dp_2}{dt} = M \tag{2.2}$$

Here the subscripts 1, 2 denote parameters referring to the channel and bow space, respectively;

$$M = \rho_1 W_1 F_1 \tag{2.3}$$

M is the mass flow rate of the gas from (or into) the channel, F_1 is the channel cross-sectional area, A is the exhaust coefficient, and W_1 is the overflow velocity. For $W_1 > 0$ the overflow is into the bow space, and for $W_1 < 0$ into the channel. For small W_1 (as compared with the speed of sound), acceleration of the stream is represented as

$$\frac{dW_1}{dt} = -\frac{1}{\rho_1}\frac{p_2 - p_1}{L} \tag{2.4}$$

where L is the characteristic dimension of the bow space.

Adding the heat conduction equation (1.5) to (2.1)-(2.4) and repeating the whole analysis carried out in section 2, we obtain that the stable combustion boundary for a continuous change in the damping decrement during passage through the critical mode is found from the solution of the equations

$$c^2 + d^2 = a_1 c - b_1 d$$
$$\chi = \frac{a_1 c - b_1 d}{\gamma(c^2 + d^2)} - \frac{\beta}{1 - \beta e \gamma^2} \tag{2.5}$$

where

$$a_1 = \nu\left(1 + \frac{r\gamma S_1}{2R_1}\right) - k\mu\frac{\gamma S_1}{2R_1}, \qquad b_1 = (\nu r - k\mu) R_1$$

χ is the ratio between the relaxation times of the channel t_1 and the heated powder layer t_2,

$$\chi = \frac{t_1}{t_2}, \qquad t_1 = \frac{V_1}{AF_*(RT_2^\circ)^{1/2}}$$

β and ε are dimensionless parameters

$$\beta=\frac{t_3}{t_2},\quad t_3=\frac{V_2}{AF_*(RT_2^\circ)^{1/2}},\quad \varepsilon=\frac{LAF_*}{t_2F_1}$$

t_3 is the relaxation time of the bow space, and the remaining notation is as before. Hence, if $\gamma=0$, the loss of stability occurs only in the case $\nu=1$. An investigation of the possibility of a jump disruption in stability leads to results analogous to [2].

3. Analysis of the Results

To study the quantitative influence of the parameters q, s, β, ε, we carried out numerical computations using (1.12) and (2.5).

The boundaries of domains for the existence of stable modes are shown in Fig. 2 in the coordinates (k, χ) for $\nu={}^2/_3$, $r=\mu=s=0$, and different q. Curves 1-3 correspond to the values $q=0$, $^1/_3$, $^2/_3$. Curve 1 has been obtained earlier in [1] for $T_2=\text{const}$. The domain of stable modes lies to the left of the appropriate curves, and a rise in q narrows the stability domain.

An analogous dependence of the stable mode boundary on the parameter q holds (Fig. 3) even for a variable surface temperature. Here the stability boundaries 1-4 obtained for $\nu={}^2/_3$, $r={}^1/_2$, $\mu=0.1$, and $s=0$ correspond to the values $q=0$, $^1/_3$, $^2/_3$, and 1. The dependence 1 for constant temperature in the chamber has been obtained earlier in [2].

The stable combustion boundaries for a change in the parameter s are presented in Fig. 4. The curves 1-3 have been constructed for $s=0$, $^1/_3$, $^2/_3$, respectively, for $\nu={}^2/_3$, $q={}^1/_3$, $r={}^1/_3$, $\mu=0.1$. An increase in the parameter s exerts a stabilizing influence on the stability.

As follows from Figs. 3 and 4, the curves approach a single vertical asymptote as $\chi\to\infty$, which corresponds to the case of constant gas pressure and temperature for an infinite chamber volume. The location of the asymptote is here independent of the values of the parameters q and s. Using the combustion stability criterion [6], an asymptotic formula for χ can be obtained from (1.12).

The quantitative influence on the combustion stability of the bow space is shown as a function of β in Figs. 5 and 6. A calculation using (2.5) showed that the influence of the parameter ε is insignificant. The fundamental results are hence presented for the mean value $\varepsilon=0.01$. Curves 1-4 in Fig. 5 have been obtained for $\beta=0$, 0.05, 0.1, and 0.5, respectively, for $\nu={}^2/_3$, $r=0$, and $\mu=0$. The dependences 1-4 in Fig. 6 have been constructed for $\beta=0$, 0.1, 0.5, and 1, respectively, for $\nu={}^2/_3$, $r={}^1/_3$, $\mu=0.1$; therefore, the presence of a bow space exerts an essentially stabilizing influence on the stability of the stationary combustion mode.

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